# Should retirement age be coupled to life expectancy? 

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#### Abstract

Increasing every year the retirement age by an amount proportional to the increase of the life expectancy gives roughly stable ratios of the number of retired to working-age people in industrialized countries. Continuous influx of immigrants, below one percent per year of the total population, is needed for this stabilization.


## KEYWORDS

Retirement, age, life expectancy.

## RESUMEN

El incrementar cada año la edad de retiro en una cantidad proporcional al incremento de la expectativa de vida produce relaciones aproximadamente estables entre el número de retirados y la población en edad laboral en países industrializados.

Un flujo continuo de inmigrantes, por abajo del uno porciento de la población total al año, es necesario para la estabilización.

## PALABRAS CLAVE

Retiro, edad, esperanza de vida.

The increase of the life expectancy (at birth) over the last centuries in the industrialized countries is enormous, as seen in figure 1 from Wilmoth's Berkeley Mortality Database for Swedish women. The nonlinearity of this increase warns us against extrapolating present trends to more than a century. Nevertheless, the increase of the number of older people is for the next few decades a rather predictable "age quake" and causes governments in the industrialized countries to plan reductions in pensions and increases in the retirement age. In France, these plans lead to social unrest in 2003, in Germany they are also discussed controversially since fall of 2002, in Brazil the legislation of 2003 was pushed


Fig. 1. Life expectancy of Swedish women, mostly from demog. berkeley.edu/ wilmoth/ mortality. The approximation by a straight line is much worse than that by a hyperbolic tangent. Thus our approximations which lead roughly to straight lines can only be used for limited times.
through against protest demonstrations of tens of thousands of public employees, in California public retirement benefits might in the future become available only after an age coupled to life expectancy. We prefer to simulate the effects of such legislation on a computer before they are imposed on millions of people (Bomsdorf ${ }^{1,2} 1993$ and 2003, Tuljapurkar et al. ${ }^{3}$ 2000, Olshansky et al. ${ }^{4}$ 2001, Laszkiewicz et al. ${ }^{5}$ 2003, Sommer and Pötzsch ${ }^{6}$ 2003).

Our basic method was described by Stauffer ${ }^{7}$ (2002) and assumes for the adult mortality function

at age $x$ a Gompertz law $\mu / \mathrm{b}=\mathrm{Ae}^{\mathrm{b}(x-\mathrm{x})}$, with timedependent parameters $\mathrm{b} \cong 0.1 ; \mathrm{X} \cong 100$, where our time unit is one year (Mildvan and Strehler ${ }^{8}$ 1960, Gavrilov and Gavrilova ${ }^{9}$ 1991, Azbel ${ }^{10}$ 1996, Wachter and Dinch ${ }^{11}$ 1997). Starting in the year 2005, immigration of people between the ages of 6 and 40 amounts to a fraction $c$ of the total population each year; $c$ is about half a percent. Births diminished drastically around 1970 to 1.4 per woman and are assumed to stay at this value, below the replacement value slightly above 2 ; thus immigration is need to stabilize the population. The results are given by $\operatorname{Stauffer}^{7}$ (2002).

Now we are interested in the ratio $R$ of the number of people beyond average retirement age $x_{r}$ to the number of working-age people and how $R$ depends on changes in this retirement age $x_{r}$. Working was assumed to start at age 20; the present retirement age was taken as 62 . Figure 2 summarizes the resulting ratio $R$ from several assumptions. The three top curves assume no immigration. The top curve assumes the retirement age to stay at 62 . The sec-ond-highest curve assumes $x_{r}$ to increase from 62 to 64 over an interval of 24 years, starting in 2011, while the third curve from top assumes an increase from 62 to 67 over the same time interval; such laws are presently discussed in Germany. The lowest curve adds half a percent immigration per year to the model given by the third-highest curve.

Such increases in $x_{r}$ are easily accepted by computers but not by humans. Public acceptance might be higher if the changes do not seem to be arbitrarily imposed by politicians but are presented as coming


Fig. 2. Ratio of retired to working-age people with retirement age changed as given in headline.
from nature, like ageing. The life expectancy $L$ at birth is a widely reported quantity, and a coupling of changes in $L$ to changes in $x_{r}$ appears more plausible and thus perhaps more acceptable. Though $L$ from cohort life tables should be better (Bomsdorf ${ }^{2}$ 2003) than $L$ from period life tables, we take $L$ as that calculated from the mortalities in the given year of the computer simulation, since this $L$ is best known to the general public. And we use both $L$ at birth and the remaining life expectancy after retirement which is more relevant for financing retirement than life expectancy at birth.

Figure 3 shows what happens if, starting after 2010, each year $x_{r}$ is increased by an amount proportional to the increase of $L$ five years earlier.


Fig. 3. Ratio of retired to working-age people with retirement age changed propotional to changes in life expectancy.


Fig. 4. Life expectancy at birth and retirement age (model of figure 3.).

(Bomsdorf ${ }^{12} 2002$ already suggested coupling to $L$ at retirement.) The proportionality factor is 1.0 for $L$ at retirement and 0.6 for $L$ at birth.

We see slight periodic oscillations not visible in figure 2, but otherwise the results look nice and show the dangerous peak in $R$ around the year 2030 to be of rather limited duration and thus perhaps better manageable than the results of figure 2. (Immigration was set at $c=0.0038$ to keep the total population constant in the second half of the 21st century). Figure 4 shows the resulting change in $L$ and $x_{r}$.

If immigration is changed into emigration at the same rate, and other parameters are unchanged compared with figure 3 , we get the ratio of figure 5 , with the population roughly halved every 14 years around the year 2100 .

In summary we found in figure3 a surprisingly stable though high ratio $R$ of pensioneers to working-


Fig. 5. As figure 3 but emigration instead if immigration; retirement couple to life expectancy at birth.

age people, if the retirement age is coupled to the life expectancy at birth. Modifications of our simulations could help developing countries like Mexico or Brazil to learn from the errors of Western Europe as make smoother the transition from a young to an aged society.

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