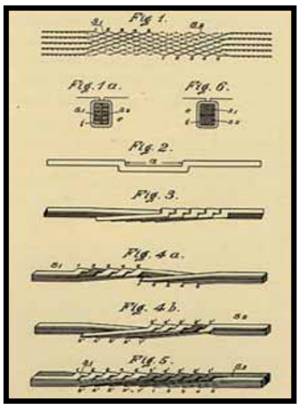
Analytical method for the optimization of Roebel bars composed of full elementary conductors

Toma Dordea, Valentin Müller, Ileana Torac,
Gheorghe Madescu, Marțian Moț, Lucian Ocolişan
Romanian Academy, Timisoara Branch, 300223 Timisoara, Bd. Mihai Viteazul nr.24
tdordea@d109lin.utt.ro , ileana_torac@yahoo.com



RESUMEN

En este trabajo se presenta un método analítico para optimizar las barras Roebel, las cuales se utilizan para los arrollamientos de los motores eléctricos potentes de corriente alterna. La construcción es mediante conductores elementales. También se determinó el campo magnético radial de la ranura y de la zona de las cabezas de la bobina.

PALABRAS CLAVE

Barras de Roebel, optimización, motores eléctricos.

ABSTRACT

An analytical method for optimizing Roebel bars is presented in this paper. These bars are employed for windings of high power, A.C. electric motors. They are built of elementary conductors. The magnetic fields in the slots and in the coil head zone was also determined.

KEYWORDS

Roebel bars, optimizations, electrical motors.

INTRODUCTION

The calculus of the Roebel bar losses is a research subject since 1970.[1, 2] A final solution allowing the bar optimization has not been found. The Roebel bar optimum structure allows increasing machine efficiency, and consequently energy savings.

Most of the employed methods estimate firstly the magnetic field using finite element (FEM) and then the winding losses.[3, 4, 5]

High precision estimation of magnetic field by FEM requires a very complex grid form and long running time. Therefore, we consider that an analytical method for the magnetic field estimation is more appropriate for optimization purposes.

The proposed analytical method uses coefficients, which are estimated using known methods, providing good accuracy.

Ludwig Roebel 1878 - 1934

THE BAR MODEL

The shape of a Roebel bar composed of full elementary conductors and that of an elementary conductor are presented in figures 1a) and 1b) respectively.

The shape of the modeled elementary conductor is presented in figure 2b). The axial length of the core is *l*, one end of the elementary conductor is marked with (') and the other end with (''). Notice that transposition is made only in the slotted area; at the end of the coil area the elementary conductors are not transposed. The distance between two successive transpositions is l_e.

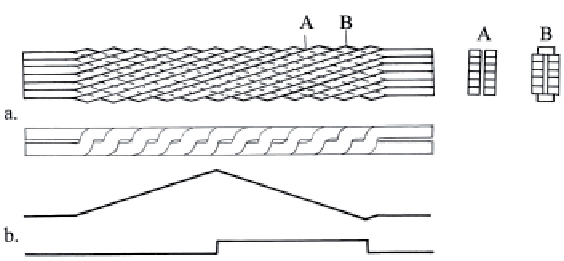


Fig. 1. The shape of a Roebel bar.

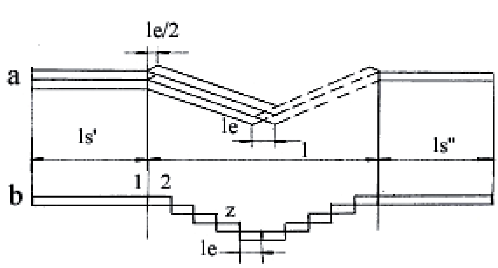


Fig. 2. The modeled elementary conductor

For the modeled bar, all the elementary conductors are transposed in the same plane z. The label *z*=1 corresponds to the frontal plane of the ferromagnetic core.

The first transposition is made in the plane *z*= 2, and the last one in the plane $z = z_f - 1$, where z_f corresponds to the ending plane of the ferromagnetic core. Every transposition changes the position of the elementary conductor in the bar structure. The elementary conductors in the plane *z*=1 are labeled from 1 to 2*m*. So, along the Roebel bar z_f-1 different bar structures can be emphasized (figure 3.) The label of a winding layer is λ. The number of the overlapped elementary conductor in a Roebel bar is *m*. There are two elementary conductors in every layer λ,: the left is labeled ε_1 and the right ε_2. Obviously, the total amount of elementary conductors is 2*m*. Notice that for ε_1 and ε_2, 2*m* is a period and for λ the period is *m*. According to these assumptions the following is obtained:

$$\varepsilon_1=\lambda+1-z \ ; \ \varepsilon_2=2(m+1)-\lambda-z$$

$$\varepsilon_1=\varepsilon_2+2\lambda-1; \ \varepsilon_2=\varepsilon_1+2(m-\lambda)+1 \qquad (1)$$

The quantities ε_1 and ε_2 have values between 1 and 2*m* and λ between 1 and m.

The following hypotheses are considered:

- The slot has a rectangular shape;
- The ferromagnetic core of the motor has an infinitely high magnetic permeability compared to that of the vacuum environment;
- All the elementary conductors have the same rectangular cross section and the same width insulation;

Z = 1

m	m+1
·	m+2
·	·
·	·
·	·
·	·
·	·
·	·
2	2m-1
1	2m

a

Z

m	m-z+1	m-z+2
·	·	·
·	·	·
·	·	·
λ	ε_1	ε_2
·	·	·
·	·	·
·	·	·
·	3-z	·
1	2-z	1-z

b

Fig. 3. The Roebel bar structure for z=1(a) and for z (b).

- In the slotted area, the magnetic field lines due to the elementary currents are perpendicular to the slot walls and parallel to the slot base.

Notice that the magnetic field in the slotted area produced by the currents crossing the elementary conductors has two parts: the inner magnetic field and the outer magnetic field. The skin effect is the result of the first one. The second part produces the leakage inductance of the bar.

The dimensions of the elementary conductors (without insulation) are b_{cu} and h_q, and that of the insulations is $\Delta_i/2$ (for a single side), the slot's width is b_c and the height of the column composed of m overlapped elementary conductors is H (figure 4).

Considering a section of the elementary conductor belonging to the layer λ, between the transposition planes z and z+1. This length is l_e. (figure 5), the area is divided along its depth in strips of thickness h. The value of h must satisfy two conditions: both h_q and Δ_i must be a multiple of h; and the current density is considered (with an acceptable error) as constant for the considered strip of thickness h. The strips are labeled beginning at the slot's base. The first strip of the h_q is $n_{\lambda I}$ and the last one is $n_{\lambda f}$. The dimension h_q has N strips. A certain strip of the layer λ is labeled ν.

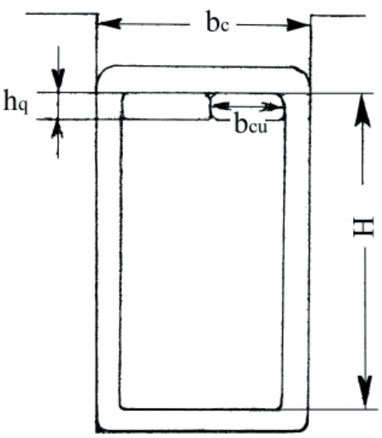


Fig. 4. Explanatory for the bar and slot's dimensions.

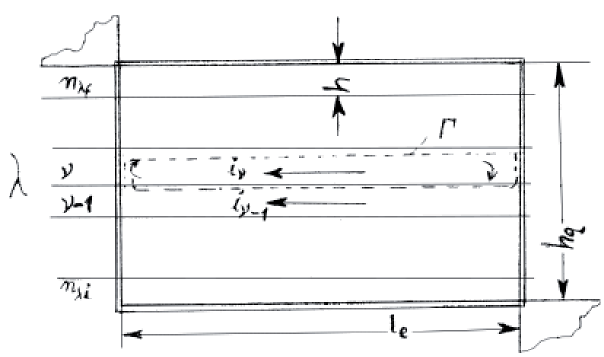


Fig. 5. Explanatory for a certain elementary domain l_e , between the transposition planes z and z+1.

Consider that the strips ν and ν-1 (figure 5). Through their upper areas take the closed curve Γ, between the planes z and z+1. The left column of the bar is marked with γ and the right one with δ. Equation (2) is obtained for Γ_γ and Γ_δ.[6, 7]

$$i_{\nu\gamma} = i_{\nu-1,\gamma} + \frac{\mu h^2}{\rho_{cu}} \frac{b_{cu}}{b_c} \frac{\mathrm{d}}{\mathrm{d}t}\left(i_{u\lambda} + \sum_{\varepsilon=n_{\lambda i}}^{\nu-1} i_{c\varepsilon} + \frac{1}{3} i_{c\nu} \right)$$

$$i_{\nu\delta} = i_{\nu-1,\delta} + i_{\nu\gamma} - i_{\nu-1,\gamma} \tag{2}$$

where μ is the magnetic permeability of the conductive material and ρ_{cu} is the electric resistivity; $i_{\nu\gamma}$, $i_{\nu\delta}$, $i_{\nu-1,\gamma}$, $i_{\nu-1,\delta}$ are the currents corresponding to the considered strip; $i_{u\lambda}$ is the sum of the currents through the elementary conductors from the first layer to the (λ-1) layer; $i_{c\varepsilon}$ is the sum of the currents $i_{\varepsilon\gamma}$ and $i_{\varepsilon\delta}$; $i_{c\nu}$ are the sum of the currents $i_{\nu\gamma}$ and $i_{\nu\delta}$ corresponding to the strip ν.

From equation (2):

$$i_{c\nu} = i_{c,\nu-1} + 2\left(i_{\nu\gamma} - i_{\nu-1,\gamma}\right) \tag{3}$$

The magnetic fields at the end of the coil area, is presented in figure 6.a.

This field is approached using the model presented in figure 6.b, were $b_c^{'} \approx b + k_c H$ (with $k_c \cong 1.2$). In this case in the corresponding relations[6,7,8] $b_c^{'}$ will be used instead of b_c .

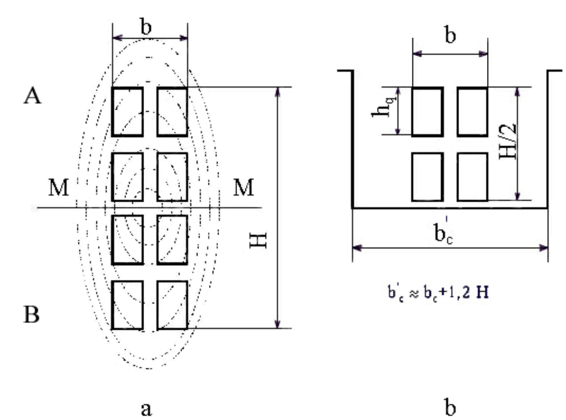


Fig. 6. Explanatory for the end coil area.
a) real field; b) equivalent model for field estimation.

THE CALCULUS OF THE CURRENTS THROUGH THE ELEMENTARY CONDUCTORS

Taking into account that the label of any elementary current is that of the elementary conductor in the plane z=1. Considering a curve Γ in the bottom side of the strip in the elementary conductors crossed by the current $\underline{I}_\xi$ and by current $\underline{I}_{\xi+1}$. Equation (4) is obtained for the curve Γ.

$$\frac{\rho_{cu} l_e}{b_c h}\sum_{z=1}^{z_f-1}\left(\underline{I}_{n\xi i}-\underline{I}_{n,\xi+1,i}\right)+\frac{\rho_{cu} l_{s'}}{b_{cu} h}\left(\underline{I}'_{n\xi i}-\underline{I}'_{n,\xi+1,i}\right)+ +\frac{\rho_{cu} l_{s''}}{b_{cu} h}\left(\underline{I}''_{n\xi i}-\underline{I}''_{n,\xi+1,i}\right)= =-\mathrm{j}\omega\left(\underline{\Psi}_{\xi\,\mathrm{int}}-\underline{\Psi}_{\xi+1,\,\mathrm{int}}\right)+\underline{U}_{e\gamma\xi}-\underline{U}_{e\gamma,\xi+1} \quad (4)$$

where $\underline{\Psi}_{\xi\mathrm{int}}$ and $\underline{\Psi}_{\xi+1,\mathrm{int}}$ are the internal fluxes of the elementary conductors ξ and ξ+1; $\underline{U}_{e\gamma\xi}$ and $\underline{U}_{e\gamma,\xi+1}$ are the induced voltages in the same elementary conductors; I is the current corresponding to the slotted area; *I'* and *I"* are the currents corresponding to the end coil areas. For the calculus of these voltages, both the radial field in the slotted area and that in the end coil area, were taken into account.

The radial magnetic field in the slotted area

The radial flux density B_{ar} in the slot domain produces circulating currents trough the elementary conductors connected in the end bar area. The value of B_{ar} depends on the position of the considered point in the slotted area. Using the conformal mapping method, the flux density in the slot axis is obtained as follows:

$$B_{rm}=B_{\delta\max}\mathrm{e}^{-\left(m_m\cdot x_h/\delta+n_m\right)} \quad (5)$$

where δ is the air gap, x_h is the coordinate along the slot axis, relative to the tooth extremity,

$\alpha_m=b_c/\delta$, and:

$$m_m=\frac{38{,}39\,\alpha_m^{\,3}-665{,}1\,\alpha_m^{\,2}+6507\,\alpha_m+1061}{\alpha_m^{\,3}+909{,}4\,\alpha_m^{\,2}+2654\,\alpha_m-1246} \quad (6)$$

$$n_m=0{,}0857\,\alpha_m+0{,}3786$$

For $\alpha_m>2$, the radial flux density distribution is a sinusoidal function for any normal to the slot axis:

$$B_{ar}=B_{rm}\sin\frac{x_b}{b_c}\pi \quad (7)$$

where x_b is the coordinate relative to the slotted wall.

In this case, the flux produced by the circulating current corresponding to the elementary length, l_e, is:

$$\Phi_{ar}=\frac{l_e b_c}{\pi}\sin\left(\frac{b_{cu}+\Delta_i}{b_c}\cdot\frac{\pi}{2}\right)B_{rm} \quad (8)$$

The emf corresponding to the layer λ, for both the adjacent conductors of the two columns, (for the length l_e), is:

$$\underline{U}_{ace\lambda}=-\mathrm{j}K_{ace}\underline{B}_{rm}\left(h_\lambda\right)\cdot\theta_\xi \quad (9)$$

where:

$$K_{ace}=\frac{f}{\pi\sqrt{2}}l_e b_c\,\sin\left(\frac{b_{cu}+\Delta_i}{b_c}\cdot\frac{\pi}{2}\right) \quad (10)$$

with $\theta_\xi=1$ for $\xi=\varepsilon_1$ and $\theta_\xi=-1$ for $\xi=\varepsilon_2$.

The resultant emf, corresponding to the total length of the slotted area, (all the elementary length l_e) is calculated as follows:

$$\underline{U}_{ar\xi}=-\mathrm{j}K_{ace}\underline{B}_{\delta\max}\sum_{z=1}^{z_f-1}\mathrm{e}^{-\left(m_m\cdot x_h/\delta+n_m\right)}\theta_\xi \quad (11)$$

The radial magnetic field in the end bar area

The end bar shape of double layer winding is usually as in figure 7a). It was considered the radial flux density ($B_{s'r}$) distribution as in figure 7b) depending essentially of the end bar shape. $B_{s'r}$ can be considered as a decreasing lineal function. In this case the induced voltage corresponding to the curve γ, which produces the circulating current in the considered elementary conductor, is (for sinusoidal conditions):

$$\underline{U}_{es'r\xi}=-\mathrm{j}\sqrt{2}\pi\,f\left(b_{cu}+\Delta_i\right)\left(l_{s'1}+\frac{1}{2}l_{s'2}\right)B_{s'r} \quad (12)$$

For the second bar extremity, the voltage is similarly $\underline{U}_{es"r\xi}$. Thus, the resultant emf that produces the circulating current $\underline{I}_\xi$ is:

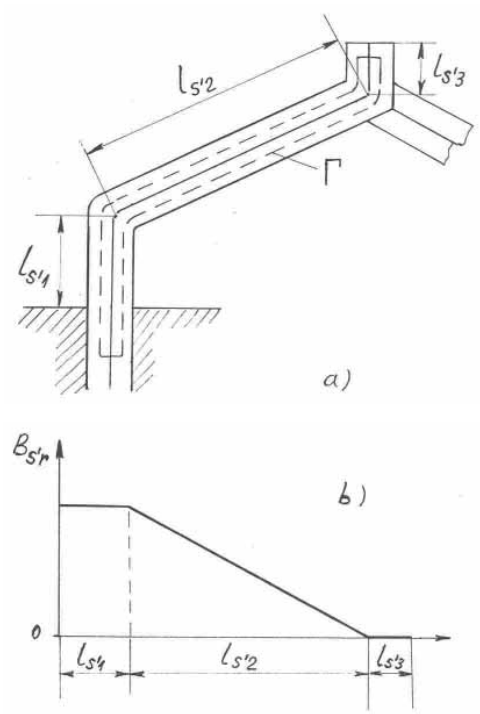


Fig. 7. Explanatory for the end coil area distribution: a) end coil dimensions; b) considered field distribution.

$$\underline{U}_{er\xi} = \underline{U}_{ar\xi} + \underline{U}_{es'r\xi} + \underline{U}_{es''r\xi} \qquad (13)$$

With equations (4) and (13) for all the $2m$ elementary conductors of the Roebel bar gives:

$$\sum_{\alpha=1}^{2m} \underline{G}(\alpha,\xi)\underline{I}_{\alpha} = \underline{G}(\xi)$$
$$\sum_{\alpha=1}^{2m} \underline{G}(\alpha,2m)\underline{I}_{\alpha} = \underline{I} \qquad (14)$$

Therefore, the elementary conductor currents and the winding losses are known.

THE ROEBEL BAR OPTIMIZATION

The most general statement of the Roebel bar optimization problem requires the following bases:

The slot dimensions are established according to a maximal use of the ferromagnetic material, the conductor width is determined based on the slot insulation conditions the total slot height H_t is available for the elementary conductors of the two bars (including the slot insulation), knowing ρ_{cu}, b_{cu}, and $H_t=H_p+H_s$. Now it is necessary to establish the number of the overlapped elementary conductors for the two bars, m_p and m_s, the bars height, H_p and H_s, the transposition numbers, z_{pf}-1 and z_{sf}-1. Notice that the last two quantities define the transposition angle.

The optimization criterion is the minimization of the bar losses, that means of the total losses corresponding to the top bar and that of the bottom bar.

A program for optimization of Roebel bar calculations was developed based on the presented method.

VALIDATION OF THE PROPOSED METHOD

The above presented method is based on the assumption of uniform current distribution on the strip. Therefore, accuracy dependents essentially of the strip height value (h).

In the case of a single conductor *(m=1)*, the A.C. resistance increasing coefficient is k_{r0}. Using the proposed strip method, the same coefficient is obtained, the value k_r. Error ε_r is defined as:

$$\varepsilon_r = 1 - k_r / k_{r0} \qquad (15)$$

The dependence of the strip method error on the strips number is presented in table I.

It was considered the case ρ_{cu} =2.16 10^{-8} Ωm, b_c= 24.2 mm, l=1.75 m, $l_{s'} = l_{s''}$ =0, HP = 66.33 mm, b_{cu}= 12.1, Δ_i =0, m = 1, f = 50 Hz, $\mu=\mu_0=4\pi\ 10^{-7}$.

The coefficient of resistance increasing (k_{r0}) is obtained as follows:

$$k_{r0} = \xi \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi} = 6.34090117 \qquad (16)$$

where:

$$\xi = HP\sqrt{\mu\omega b_{cu} / (2\rho_{cu} b_c)} = 6.340761 \qquad (17)$$

Table I. Variation of the method error on the strip´s number.

n	k_r	ε_r
500	6.31329986	4.35 10^{-3}
10^3	6.32690805	2.2 10^{-3}
10^4	6.33948738	2.22 10^{-4}
10^5	6.34076100	2.21 10^{-5}
10^6	6.34088851	1.99 10^{-6}
9.4 10^6	6.34090117	0

Hence, the necessary strip amount is calculated for an initially accepted error, practically, $n=10^5$ allows a very good estimate accuracy.

EXAMPLE

An 216 MVA, 15 kV, 50 Hz hydro generator, the phase current I_f=7910 A, p=42pole pairs, a=3, l=1.75 m, b_c=24.2 mm, b_{cu}=7.5 mm, Δ_i=0.21 mm, $m_p = m_s$=33 layers for each column, the transposition angle 2π, $z_{pf} = z_{sf}$ = 67, $H_p = H_s$ = 66.33 mm

Case A

For the maximum value of the air gap flux density B_δ=0.9T, the radial flux density at the end coil area B_r=0.4T, the results are as follows:

The losses in the first bar (P): P_{cup} = 835.426 W

The losses in the second bar (S): P_{cus}=926.593 W

The total losses for two bars: P_{cut}=1762.019 W

The DC bar losses are: P_{cu0}=452.9 W

Then the losse coefficients $k_{pcup}=P_{cup}/P_{cu0}$ =1.84451, and $k_{pcus}= P_{cus}/P_{cu0}$ =2.04579 for the bar P and S respectively.

Case B

If the two bar's height is $H_p=H_s$=66.33 mm and $z_{pf}=z_{sf}$=64, then P_{cup}=791.68 W, P_{cus}=911.60 W, P_{cut}=1703.28 W.

Case C

Taking as variable quantities z_{pf}, z_{sf} and m_p, m_s for the same height of the two bars.

Using the optimization program gives the optimal solution for m_p= 26, m_s=46, z_{pf}=47, z_{sf}= 90, with P_{cup}=747.27W, P_{cus}=858.472 W, P_{cut}=1595.742 W.

The difference between the winding losses corresponding to the non-optimized machine and that of the optimized one is ΔP_{ma}. The difference of energy losses corresponding to optimized motor against the initial solution (A) is ΔW_{et}. It was considered a period of 25 years of motor service. ΔC is the loss cost in Euro (considering 100Euro/MWh). The results are presented in table II.

Using the proposed analytical method for analyzing the influence of bar dimensions (bar's height, layers number, transposition's number) on the windings losses.

Table II. Optimization results.

Case	ΔP_{ma}. [kW]	ΔW_{et} [GWh]	ΔC [Euro]
B	44.4	9.724	972400
C	125.7	27.528	2752800

Assuming as reference case that of B_δ=0 T, B_r=0 T,. the defined coefficient losses for the lower bar (P), for the upper one (S) and for both two bars of the slot are: $k_{p1}=P_{cup}/P_{cu0}$, $k_{s1}=P_{cus}/P_{cu0}$, $k_{sc}=P_{cu}/P_{cu0}$, P_{cu0} are the bar losses for the case m=1.

The coefficient losses for the bottom bar (P), for the top bar (S), and for both slot bars are KPP, KPS, KPC, respectively. These quantities, optimized for a transposition angle value around 2π and 3π and for $B_\delta = 0$ and $B_r = 0$ are presented in figures 8a) and 8b) respectively. Notice that each point of the presented curve corresponds to an optimal motor. These solutions have different values for the elementary conductors number of the bar column (MP and MS are the elementary conductor numbers of the bar P and S respectively), and for the transposition number (ZPF and ZPS correspond to the bar P and S respectively), and the same slot height value.

The purpose of the minimum losses requires a losses coefficient KPC ≅1. For both cases of a transposition angle of 2π and 3π the curves have a minimum value for HP>HS. But KPS>KPP, so it is

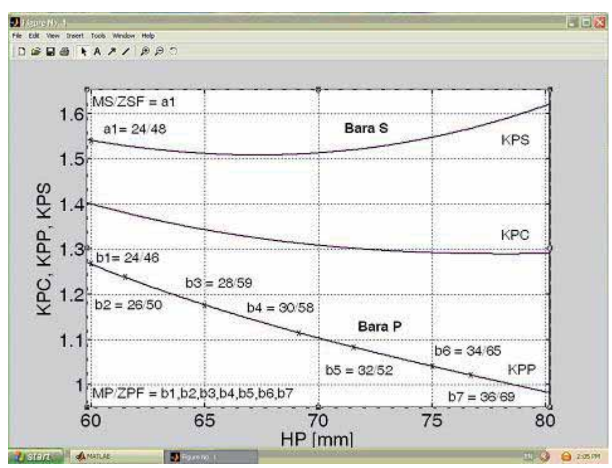


Fig. 8. a. Optimization results for B_δ=0, B_r=0, β=2π where B1(MP/ZPF=24/46), B2(26/50), B3(28/59), B4(30/58), B5(32/52), B6(34/65), B7(36/69).

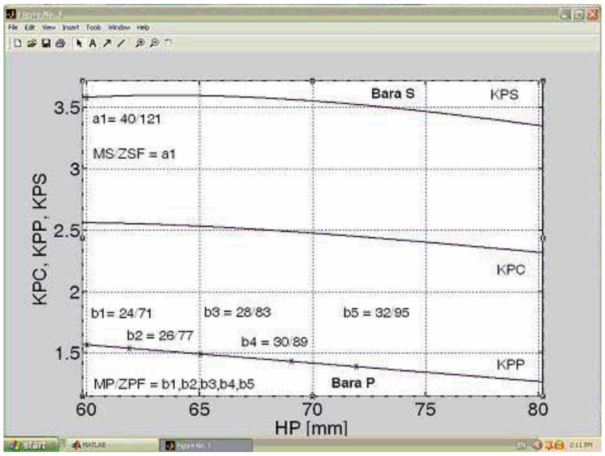


Fig. 8. b. Optimization results for B_δ=0, B_r=0, β =3π where B1(MP/ZPF=24/71), B2(26/77), B3(28/83), B4(30/89), B5(32/95).

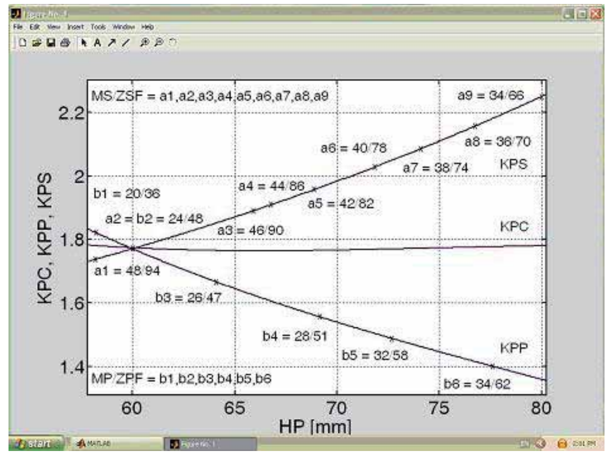


Fig. 9. a. Optimization results for B_δ=0.9T, Br=0.4T, β=2π where A1 (MS/ZSF=48/94), A2(24/48), A3(46/90), A4(44/86), A5(42/82), A6(40/78), A7(38/74), A8(36/70), A9(34/66) B1 MP/ZPF=20/36), B2(24/48), B3(26/47), B4(28/51), B5(32/58), B6(34/62).

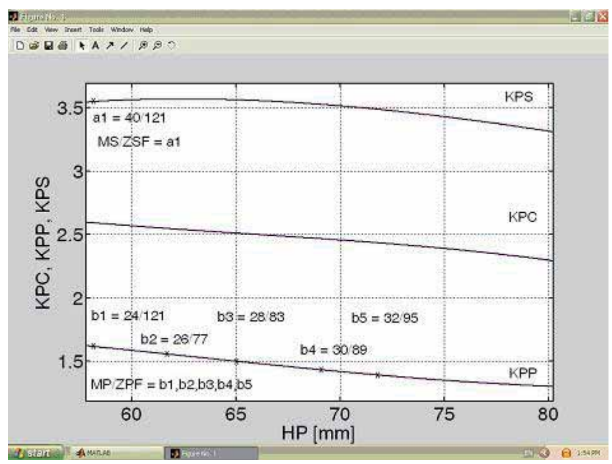


Fig. 9. b. Optimization results for B_δ=0.9T, B_r=0.4T, β =3 π, where B1(MS/ZF=24/71), B2(26/77), B3(28/83), B4(30/89), B5(32/95).

necessary to take into account the thermal conditions. Practically, because the top bar's height is limited by thermal conditions, the optimum solution is also defined by the same thermal limits.

A domain around a transposition angle value of 2π and 3π was analyzed. Both cases – without radial magnetic field and with its influence – have the best solution around a transposition angle value of 2π. The cases obtain the lowest values around z_f .

Figure 9 presents the coefficients KPP, KPS, KPC for B_δ=0.9 T, B_r=0.4 T and a transposition angle 2π (figure 9a) and 3π (figure 9b).

If the radial field is taken in to account, KPC=1.762 for β=2π, and KPC=2.515 for β=3π. Neglecting the radial field, the optimum transposition angle is exactly 2π. Considering the radial field at the end coils areas, the optimal transposition angle is less than 2π.

The winding losses are dependent on the ratio between the end coil length and the axial length of the ferromagnetic core.

CONCLUSION

The analytical method based on strips theory allows a faster analysis of the structure bar influence on winding losses.

The estimation error value decreases as the strip number increase. Notice that better accuracy implies longer computer time.

This paper emphasizes that the best transposition angle value is around 2π. The ratio between the end coils length and the slotted area bar length influences the winding losses.

The proposed analytical method is very useful at medium and high power electrical motors optimization.

REFERENCES

1. Macdonald D.C., "Losses in Roebel bars: effect of slot position on circulating currents", *Proceedings IEE,* vol. 117, No.1, January 1970.

2. Macdonald D.C., “Circulating current loss within Roebel bar stator windings in hydroelectric alternators”, *Proceedings IEE*, Vol. 118, No.5 May 1971.
3. Schwery A., Traxler-Samek G., Schmidt E.,“ Numerical and Analytical Computation Methods for The Refurbishment of the Hydro-Generators”, *Proceedings of ICEM 2002*, Brugge-Belgium, 25-28 august 2002.
4. Haldemann J., „Transposition in Stator Bars of Large Turbogenerators”, *IEEE Transactions on Energy Conversion,* vol. 19, nr.3, sept, 2004, pp.555-560.
5. Iseli M., Reichert K., Neidhorfer G., “End Region Field and Circulating Currents in Roebel bars” ICEM 1990, vol. II, p.718-723.
6. T. Dordea, M. Biriescu, Ghe. Madescu, Ileana Torac, M. Moţ, L. Ocolişan, “Calcul des courents electriques par les conducteurs elementaires d’une barre Roebel. Part I: Fondements de calcul”, *Rev. Roum. Sci. Techn. – Électrotechn. et. Énerg.*, vol. 48, nr. 2-3, pp. 359-368, Bucarest, 2003;.
7. T. Dordea, M. Biriescu, Ghe. Madescu, Ileana Torac, M. Moţ, L. Ocolişan, “Calcul des courents electriques par les conducteurs elementaires d’une barre Roebel. Part II: Determination des courents” *Rev. Roum. Sci. Tech. – Électrotechn. et Énerg.*, 49, nr.1, pp.3-29, Bucarest, 2004
8. T. Dordea, Ghe. Madescu, Ileana Torac, M. Moţ, L. Ocolişan –The current distribution on the elementary conductors of the Roebel bar-Theoretical basis. Proceeding of the OPTIM 2004, Brasov, Romania
9. K. Takahashi, K.Ide, M.Onoda, K.Hattori, M. Sato, M.Takahashi, „Strand Current Distribution of Turbine Generator Full-Scale Model Coil”, *Proceedings of ICEM 2002*, Brugge-Belgium, 25-28 august 2002.