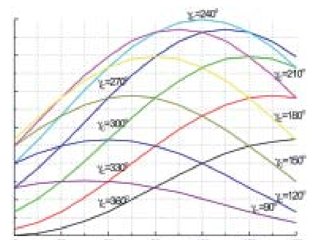
Analysis of the unsymmetrical induction motor supplied by unbalanced voltage system

Gheorghe Madescu, Marius Biriescu, Marţian Moţ, Valentin Müller
Romanian Academy, Timişoara Branch
gmadescu@d109lin.utt.ro , martian@d109lin.utt.ro

ABSTRACT

An induction motor with unsymmetrical stator windings fed by three-phase unbalanced voltages is analyzed. It is a highly generalized case, where the method of the symmetrical components is applied. The symmetrical components of voltages and currents were determined, as well as the expressions of motor's positive, negative and zero sequence impedances. The obtained results allow the steady-state analysis of three-phase induction motors built with distinct types or values of stator windings asymmetry.



KEYWORDS

Unsymmetrical induction motor, symmetrical components.

RESUMEN

El trabajo analiza el motor de inducción trifásico con embobinados de estator no simétricos, alimentados por un sistema trifásico con voltajes desequilibrados. Se trata de un caso de un alto grado de generalización en que se aplica el método de los componentes simétricos. Se han determinado los componentes simétricos de las tensiones y corrientes así como las impedancias para secuencia positiva, negativa y cero. Los resultados obtenidos permiten el análisis, en régimen estacionario, de los motores de inducción trifásicos fabricados con distintos tipos o grados de asimetría en los embobinados del estator.

PALABRAS CLAVE

Motor de inducción no simétrico, componentes simétricos.

INTRODUCTION

Indisputably, one of the most powerful method for the analysis of the electrical machines under conditions of unbalanced voltage supply is the method of symmetrical components, invented by C.L. Fortescue.[1,2] This method is widely used in the analysis of unbalanced static networks, but it is most appropriate for the analysis of symmetrical machines during unbalanced operations.

Fortescue's method has been extended by different authors,[3,4,5,6] to cover certain types of single-phase motors. The basis of the symmetrical component method is to split the single system of unbalanced voltages into two or more independent systems of balanced voltages.[7] According to this approach, the three-phase system of unbalanced voltages can be split into three symmetrical components namely: positive, negative and zero sequence components.

This paper presents an application of the method of symmetrical components in a more general case than in those presented in technical literature. This application brings up the issue of a three-phase induction motor with unsymmetrical stator phases, supplied by a three-phase system of unbalanced voltages. The paper includes the calculation of the symmetrical components of positive, negative and zero sequence current and voltages, and the determination of the "symmetrical components" of motor impedances.

THE MAGNETIC FIELD IN THE AIR GAP

It is considered a three-phase induction motor with unsymmetrical phases (figure 1). It is presumed that stator windings are distributed sinusoidally in space. The phase windings A, B and C have w_A, w_B and respectively w_C turns. It is considered that $\gamma_B \neq \gamma_C \neq 0$. Stator windings are supplied in steady-state sinusoidal conditions by a three-phase unbalanced voltage system: u_A, u_B, u_C. Phase currents:

$$i_A = \sqrt{2}\, I_A \cos\omega_1 t$$
$$i_B = \sqrt{2}\, I_B \cos(\omega_1 t - \varphi_B)$$
$$i_C = \sqrt{2}\, I_C \cos(\omega_1 t - \varphi_C), \qquad (1)$$

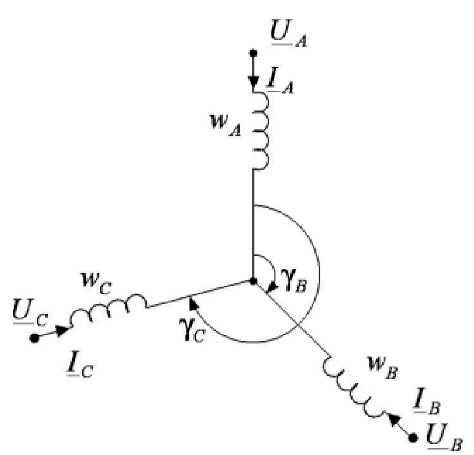


Fig. 1. Schematic representation of the unsymmetrical three-phase induction motor; the rotor has a symmetrical cage.

flowing in stator windings are expressed in the complex plane as:

$$\underline{I}_A = I_A \cdot e^{-j0}$$
$$\underline{I}_B = I_B \cdot e^{-j\varphi_B}$$
$$\underline{I}_C = I_C \cdot e^{-j\varphi_C}. \qquad (2)$$

For each phase it is considered a sinusoidal variation of the magnetic field in the air gap along the pole pitch. The magnetic fields have maximum values in pole axes and for effective value of the magnetizing force there are considered expressions:

$$\theta_A = \frac{4}{\pi} w_A \cdot k_{wA} \cdot i_A$$
$$\theta_B = \frac{4}{\pi} w_B \cdot k_{wB} \cdot i_B$$
$$\theta_C = \frac{4}{\pi} w_C \cdot k_{wC} \cdot i_C, \qquad (3)$$

where k_{w_A}, k_{w_B}, k_{w_C} represents the winding factors of the three phases. Under complex form:

$$\underline{\theta}_A = \frac{4\sqrt{2}}{\pi} w_A \cdot k_{wA} \cdot \underline{I}_A$$
$$\underline{\theta}_B = \frac{4\sqrt{2}}{\pi} w_B \cdot k_{wB} \cdot \underline{I}_B$$
$$\underline{\theta}_C = \frac{4\sqrt{2}}{\pi} w_C \cdot k_{wC} \cdot \underline{I}_C. \qquad (4)$$

If the effects of the three phase fields are added in the air gap, then a space vector of the magnetizing force is obtained under the following expression:

$$\vec{\theta} = \theta_A + \theta_B \cdot e^{j\gamma_B} + \theta_C \cdot e^{j\gamma_C}. \qquad (5)$$

The space vector defined by relation (5) includes also the variation in time of the units for each phase and the variation in space of their resulting sum. This vector is a complex quantity from a complex plane perpendicular to the motor axis. The orthogonal components of this space vector are:

$$\vec{\theta} = \theta_d + j\theta_q, \qquad (6)$$

to which it is added the zero component:

$$\theta_0 = \frac{1}{3}(\theta_A + \theta_B + \theta_C). \qquad (7)$$

In the analysis below the method of symmetrical components shall be used. It is known that a sinusoidal time function may be expressed as a sum of two conjugated complex quantities. As a result the following may be stated:

$$\theta_A = \frac{1}{2}\left(\underline{\theta}_A + \underline{\theta}_A^*\right)$$
$$\theta_B = \frac{1}{2}\left(\underline{\theta}_B + \underline{\theta}_B^*\right) \qquad (8)$$
$$\theta_C = \frac{1}{2}\left(\underline{\theta}_C + \underline{\theta}_C^*\right)$$

With the expressions (8) and (5) it is obtained in the end:

$$\vec{\theta} = \frac{1}{2}\left(\underline{\theta}_A + \underline{\theta}_B \cdot e^{j\gamma B} + \underline{\theta}_c \cdot e^{j\gamma C}\right) + $$
$$+ \frac{1}{2}\left(\underline{\theta}_A + \underline{\theta}_B \cdot e^{-j\gamma B} + \underline{\theta}_c \cdot e^{-j\gamma C}\right) = \vec{\theta}_+ + \vec{\theta}_- \qquad (9)$$

where $\vec{\theta}_+$ is a space vector of positive sequence and $\vec{\theta}_-$ is a space vector of negative sequence.

Generally, $\vec{\theta}$ represents the magnetizing force of an elliptical rotating magnetic field, $\vec{\theta}_+$ and $\vec{\theta}_-$ represent the magnetizing forces of two circular magnetic fields turning in opposite directions, produced by two three-phase symmetrical current systems.

THE SYMMETRICAL COMPONENTS OF CURRENTS

We shall obtain in this paragraph the relations to calculate each current component, when the values of the currents ($\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$) through the motor phases are known.

The three-phase current system of positive sequence ($\underline{I}_{A+}$, $\underline{I}_{B+}$, $\underline{I}_{C+}$) has to produce in the air gap only one space vector of positive sequence ($\vec{\theta}_+$). The negative sequence field ($\vec{\theta}_-$) and the zero sequence field (θ_0) have to be zero.

As a result, following conditions are obtained from relations (7) and (9):

$$\frac{1}{2}\left(\underline{\theta}_{A+} + \underline{\theta}_{B+} \cdot e^{j\gamma B} + \underline{\theta}_{C+} \cdot e^{j\gamma C}\right) = \frac{3}{2}\theta_{A+}$$
$$\frac{1}{2}\left(\underline{\theta}_{A+} + \underline{\theta}_{B+} \cdot e^{-j\gamma B} + \underline{\theta}_{C+} \cdot e^{-j\gamma C}\right) = 0$$
$$\frac{1}{3}\left(\underline{\theta}_{A+} + \underline{\theta}_{B+} + \underline{\theta}_{C+}\right) = 0 \qquad (10)$$

Using relation (4) and coefficients

$$k_B = \frac{w_B \cdot k_{wB}}{w_A \cdot k_{wA}} \neq 0\,;\; k_C = \frac{w_C \cdot k_{wC}}{w_A \cdot k_{wA}} \neq 0\,, \qquad (11)$$

the equation system (10) becomes:

$$-2\underline{I}_{A+} + k_B \underline{I}_{B+} \cdot e^{j\gamma B} + k_C \underline{I}_{C+} \cdot e^{j\gamma C} = 0$$
$$\underline{I}_{A+} + k_B \underline{I}_{B+} \cdot e^{-j\gamma B} + k_C \underline{I}_{C+} \cdot e^{-j\gamma C} = 0 \qquad (12)$$
$$\underline{I}_{A+} + k_B \underline{I}_{B+} + k_C \underline{I}_{C+} = 0$$

This system of linear equations is homogenous and allows determination of fictitious currents of positive sequence. In order for a solution to exists, the determinant of the equations system must be zero. If this condition is fulfilled, from (12) the following solutions are obtained in the end:

$$\underline{I}_{B+} = \frac{1}{k_B}\underline{I}_{A+} \cdot a^2\,;\; \underline{I}_{C+} = \frac{1}{k_C}\underline{I}_{A+} \cdot a\,, \qquad (13)$$

where

$$a = e^{j\frac{2\pi}{3}}\,;\; a^2 = e^{j\frac{4\pi}{3}}. \qquad (14)$$

Following similar considerations, for the three-phase current system of negative sequence ($\underline{I}_{A-}$, $\underline{I}_{B-}$, $\underline{I}_{C-}$), we obtain the solutions:

$$\underline{I}_{B-} = \frac{1}{k_B}\underline{I}_{A-} \cdot a\,;\; \underline{I}_{C-} = \frac{1}{k_C}\underline{I}_{A-} \cdot a^2. \qquad (15)$$

The zero component of the magnetizing force produces in the air gap an alternative magnetic field. As a result, it should be imposed the condition that the two fields (positive and negative) have equal magnitudes and in this way, by superposition it should be obtained a single alternative magnetic field, with fixed direction in space. In consequence, the condition is stated as:

$$\left|\underline{I}_{A0} + k_B \underline{I}_{B0} \cdot e^{j\gamma B} + k_C \underline{I}_{C0} \cdot e^{j\gamma C}\right| =$$
$$= \left|\underline{I}_{A0} + k_B \underline{I}_{B0} \cdot e^{-j\gamma B} + k_C \underline{I}_{C0} \cdot e^{-j\gamma C}\right|. \qquad (16)$$

From this condition the currents are given by:

$$\underline{I}_{B0} = \frac{1}{k_B}\underline{I}_{A0}\,;\; \underline{I}_{C0} = \frac{1}{k_C}\underline{I}_{A0}. \qquad (17)$$

Using the above relations, at this time the transfer could be made from phase components to the symmetrical ones as well as the reverse transfer.

Using the methods of symmetrical components, and the relations (13), (15), (17), we may write:

$$\begin{aligned} \underline{I}_A &= \underline{I}_{A+} + \underline{I}_{A-} + \underline{I}_{A0} \\ \underline{I}_B &= \frac{1}{k_B}\underline{I}_{A+}\mathrm{a}^2 + \frac{1}{k_B}\underline{I}_{A-}\mathrm{a} + \frac{1}{k_B}\underline{I}_{A0} \\ \underline{I}_C &= \frac{1}{k_C}\underline{I}_{A+}\mathrm{a} + \frac{1}{k_C}\underline{I}_{A-}\mathrm{a}^2 + \frac{1}{k_C}\underline{I}_{A0} \end{aligned} \quad (18)$$

Under matricial form system (18) becomes:

$$[\underline{I}] = [A][\underline{I}_S], \quad (19)$$

where have been used notations:

$$[\underline{I}] = \begin{bmatrix} \underline{I}_A \\ k_B\underline{I}_B \\ k_C\underline{I}_C \end{bmatrix}; \quad [A] = \begin{bmatrix} 1 & 1 & 1 \\ \mathrm{a}^2 & \mathrm{a} & 1 \\ \mathrm{a} & \mathrm{a}^2 & 1 \end{bmatrix}; \quad [\underline{I}_S] = \begin{bmatrix} \underline{I}_{A+} \\ \underline{I}_{A-} \\ \underline{I}_{A0} \end{bmatrix}. \quad (20)$$

THE SYMMETRICAL COMPONENTS OF VOLTAGES

Fictitious currents of positive, negative and zero sequence are produced by fictitious voltages of positive, negative and zero sequence.

According to the methods of symmetrical components and the conservation criterion of complex powers we may write:

$$\begin{aligned} \underline{U}_A &= \underline{U}_{A+} + \underline{U}_{A-} + \underline{U}_{A0} \\ \underline{U}_B &= k_B\underline{U}_{A+}\mathrm{a}^2 + k_B\underline{U}_{A-}\mathrm{a} + k_B\underline{U}_{A0} \\ \underline{U}_C &= k_C\underline{U}_{A+}\mathrm{a} + k_C\underline{U}_{A-}\mathrm{a}^2 + k_C\underline{U}_{A0}. \end{aligned} \quad (21)$$

Under matricial form the last system becomes:

$$[\underline{U}] = [A][\underline{U}_S], \quad (22)$$

where:

$$[\underline{U}] = \begin{bmatrix} \underline{U}_A \\ \frac{1}{k_B}\underline{U}_B \\ \frac{1}{k_C}\underline{U}_C \end{bmatrix}; \quad [\underline{U}_S] = \begin{bmatrix} \underline{U}_{A+} \\ \underline{U}_{A-} \\ \underline{U}_{A0} \end{bmatrix}. \quad (23)$$

THE IMPEDANCES OF THE SINGLE-PHASE INDUCTION MOTOR

If an induction motor has only one winding in the stator, a single-phase motor is obtained. Considering the three-phase induction motor (figure 1) having only one phase A supplied by a sinusoidal voltage, we obtain also the case of a single-phase motor. It is known that under these conditions, the alternative magnetic field in the air gap may be split in two symmetrical rotating fields, which rotate in opposite directions (of positive sequence and of negative sequence).

The positive sequence field induces in the stator winding an e.m.f. of positive sequence (E+) and the negative sequence field induces an e.m.f. of negative sequence (E-). As to these two fields, the single-phase motor winding has two impedances: a positive sequence impedance (Z+) in relation to the positive sequence field and negative sequence impedance (Z-) in relation to the negative sequence, respectively.

The equivalent circuit (figure 2) corresponding to the single-phase motor with symmetrical cage-rotor and the values of such impedances are known:

$$\begin{aligned} \underline{Z}_+ &= \frac{1}{2}\cdot\frac{\mathrm{j}X_m\left(\frac{R_2'}{s} + \mathrm{j}X_{2\sigma}'\right)}{\frac{R_2'}{s} + \mathrm{j}\left(X_{2\sigma}' + X_m\right)} \\ \underline{Z}_- &= \frac{1}{2}\cdot\frac{\mathrm{j}X_m\left(\frac{R_2'}{2-s} + \mathrm{j}X_{2\sigma}'\right)}{\frac{R_2'}{2-s} + \mathrm{j}\left(X_{2\sigma}' + X_m\right)} \end{aligned} \quad (24)$$

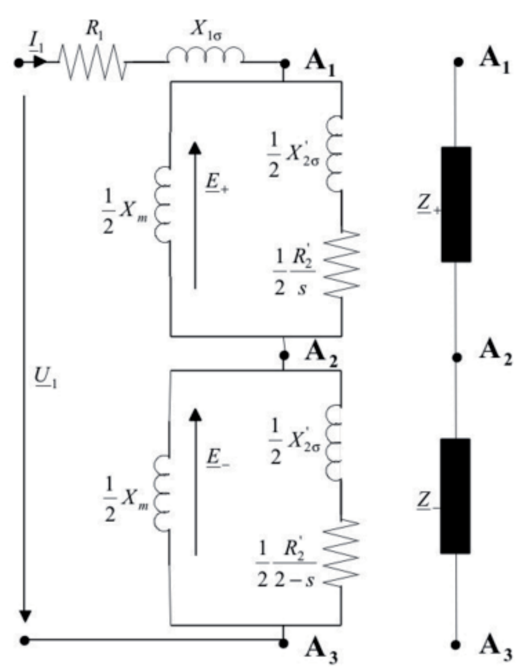


Fig. 2. The equivalent circuit of the single-phase motor.

$$R_2^{'} = k_t R_2 \,;\; X_{2\sigma}^{'} = k_t X_{2\sigma} \,,$$

$$k_t = \frac{m_1 \left(w_1 k_{w1}\right)^2}{m_2 \left(w_2 k_{w2}\right)^2} = \frac{8\left(w_1 k_{w1}\right)^2}{N_2}$$

where, for single phase motor with symmetrical cage rotor was considered:[8,9] $m_1=2$; $m_2=N_2$ (rotors bars number); $w_2=1/2$; $k_{w2}=1$.

In the relations further on, the parameters defined above are used.

THE IMPEDANCES OF THE THREE-PHASE SYMMETRICAL INDUCTION MOTOR [7,10,11]

If the three windings of the motor are symmetrical ($\gamma_B = 2\pi/3, \gamma_C = 4\pi/3$), then in the air gap exists only a positive sequence of the magnetic field and in consequence a positive sequence impedance.

In this case, the equivalent circuit with parameters previously defined in relations (24) for the single phase motor is presented in figure 3. In the phase equivalent circuit, between two stator phases appears additionally a mutual leakage reactance $X_{11\sigma}$.

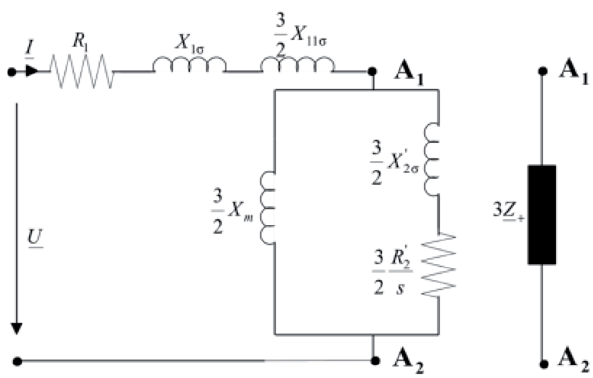


Fig. 3. The phase equivalent circuit of the three-phase symmetrical induction motor.

THE IMPEDANCES OF THE THREE-PHASE UNSYMMETRICAL INDUCTION MOTOR

Let us consider now the three-phase motor from figure 1, supplied by a three-phased unbalanced voltage system. In such condition, in the air gap of the motor are to be found six rotating magnetic fields, two for each motor phase. Three fields rotate in positive direction (forward wave) and other three in reverse direction (backward wave). Each of such fields induces an e.m.f. in stator phases. For phase A, the six e.m.f. have the expressions:

$$\begin{aligned} &\underline{E}_{+A} = -\underline{Z}_{+}\underline{I}_{A}\,;\,\underline{E}_{-A} = -\underline{Z}_{-}\underline{I}_{A} \\ &\underline{E}_{+AB} = -k_B \underline{Z}_{+}\,\mathrm{e}^{\mathrm{j}\gamma_B}\,\underline{I}_{B}\,;\,\underline{E}_{-AB} = -k_B \underline{Z}_{-}\,\mathrm{e}^{-\mathrm{j}\gamma_B}\,\underline{I}_{B} \\ &\underline{E}_{+AC} = -k_C \underline{Z}_{+}\,\mathrm{e}^{\mathrm{j}\gamma_C}\,\underline{I}_{C}\,;\,\underline{E}_{-AC} = -k_C \underline{Z}_{-}\,\mathrm{e}^{-\mathrm{j}\gamma_C}\,\underline{I}_{C} \end{aligned} \qquad (25)$$

where Z+ and Z- are the impedances from the relation (24).

Due to the mutual leakage inductance, in phase A are also induced following e.m.f. caused by phase B and phase C respectively:

$$\begin{aligned} &\underline{E}_{\sigma AB} = -\,\mathrm{j}\,k_B X_{AA\sigma} \cos\gamma_B\,\underline{I}_B \\ &\underline{E}_{\sigma AC} = -\,\mathrm{j}\,k_C X_{AA\sigma} \cos\gamma_C\,\underline{I}_C\,. \end{aligned} \qquad (26)$$

As a result, for phase A it is obtained the diagram in figure 4, where:

$$\underline{U}_A = \underline{Z}_{AA}\underline{I}_A + \underline{Z}_{AB}\underline{I}_B + \underline{Z}_{AC}\underline{I}_C\,, \qquad (27)$$

where:

$$\begin{aligned} &\underline{Z}_{AA} = R_A + \mathrm{j}\,X_{A\sigma} + \mathrm{j}\,X_{AA\sigma} + \underline{Z}_{+} + \underline{Z}_{-} \\ &\underline{Z}_{AB} = k_B\left(\underline{Z}_{+}\mathrm{e}^{\mathrm{j}\gamma_B} + \underline{Z}_{-}\mathrm{e}^{-\mathrm{j}\gamma_B} + \mathrm{j}\,X_{AA\sigma}\cos\gamma_B\right) \\ &\underline{Z}_{AC} = k_C\left(\underline{Z}_{+}\mathrm{e}^{\mathrm{j}\gamma_C} + \underline{Z}_{-}\mathrm{e}^{-\mathrm{j}\gamma_C} + \mathrm{j}\,X_{AA\sigma}\cos\gamma_C\right). \end{aligned} \qquad (28)$$

Similarly, for phases B and C it may be written:

$$\begin{aligned} &\underline{U}_B = \underline{Z}_{BA}\underline{I}_A + \underline{Z}_{BB}\underline{I}_B + \underline{Z}_{BC}\underline{I}_C \\ &\underline{U}_C = \underline{Z}_{CA}\underline{I}_A + \underline{Z}_{CB}\underline{I}_B + \underline{Z}_{CC}\underline{I}_C\,, \end{aligned} \qquad (29)$$

with impedances:

$$\begin{aligned} &\underline{Z}_{BA} = k_B\left(\underline{Z}_{+}\mathrm{e}^{-\mathrm{j}\gamma_B} + \underline{Z}_{-}\mathrm{e}^{\mathrm{j}\gamma_B} + \mathrm{j}\,X_{AA\sigma}\cos\gamma_B\right) \\ &\underline{Z}_{BB} = R_B + \mathrm{j}\,X_{B\sigma} + k_B^2\left(\mathrm{j}\,X_{AA\sigma} + \underline{Z}_{+} + \underline{Z}_{-}\right) \\ &\underline{Z}_{BC} = k_B k_C\left(\underline{Z}_{+}\mathrm{e}^{\mathrm{j}(\gamma_C-\gamma_B)} + \underline{Z}_{-}\mathrm{e}^{-\mathrm{j}(\gamma_C-\gamma_B)} + \right. \\ &\qquad \left. +\mathrm{j}\,X_{AA\sigma}\cos\left(\gamma_C - \gamma_B\right)\right) \\ &\underline{Z}_{CA} = k_C\left(\underline{Z}_{+}\mathrm{e}^{-\mathrm{j}\gamma_C} + \underline{Z}_{-}\mathrm{e}^{\mathrm{j}\gamma_C} + \mathrm{j}\,X_{AA\sigma}\cos\gamma_C\right) \\ &\underline{Z}_{CB} = k_B k_C\left(\underline{Z}_{+}\mathrm{e}^{-\mathrm{j}(\gamma_C-\gamma_B)} + \underline{Z}_{-}\mathrm{e}^{\mathrm{j}(\gamma_C-\gamma_B)} + \right. \\ &\qquad \left. +\mathrm{j}\,X_{AA\sigma}\cos\left(\gamma_C - \gamma_B\right)\right) \\ &\underline{Z}_{CC} = R_C + \mathrm{j}\,X_{C\sigma} + k_C^2\left(\mathrm{j}\,X_{AA\sigma} + \underline{Z}_{+} + \underline{Z}_{-}\right) \end{aligned} \qquad (30)$$

As matrix, the equations (27) and (29) are written:

$$\left[\underline{U}\right] = \left[\underline{Z}\right]\left[\underline{I}\right], \qquad (31)$$

where:

$$[\underline{Z}] = \begin{bmatrix} \underline{Z}_{AA} & \frac{1}{k_B}\underline{Z}_{AB} & \frac{1}{k_C}\underline{Z}_{AC} \\ \frac{1}{k_B}\underline{Z}_{BA} & \frac{1}{k_B^2}\underline{Z}_{BB} & \frac{1}{k_B k_C}\underline{Z}_{BC} \\ \frac{1}{k_C}\underline{Z}_{CA} & \frac{1}{k_B k_C}\underline{Z}_{CB} & \frac{1}{k_C^2}\underline{Z}_{CC} \end{bmatrix} = 0 \quad .(32)$$

If, according to the relations of transformation in equation (31), symmetrical components are used, we obtain:

$$[A][\underline{U}_s] = [\underline{Z}][A]\,[\underline{I}_s] \quad (33)$$

or:

$$[\underline{U}_S] = [A]^{-1}[\underline{Z}][A]\,[\underline{I}_S]. \quad (34)$$

The last equation is written with symmetrical components. In this system the following symmetrical motor impedances are defined:

Positive sequence impedance: $\underline{Z}_{A_+} = \dfrac{\underline{U}_{A_+}}{\underline{I}_{A_+}}$;

Negative sequence impedance: $\underline{Z}_{A_-} = \dfrac{\underline{U}_{A_-}}{\underline{I}_{A_-}}$;

Zero sequence impedance: $\underline{Z}_{A_0} = \dfrac{\underline{U}_{A_0}}{\underline{I}_{A_0}}$.

As a result, we may write the following equation of voltages as a matrix:

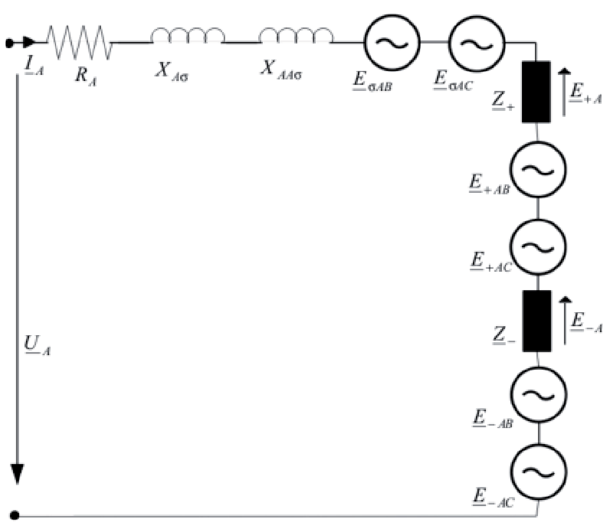


Fig. 4. The equivalent circuit for phase A of the unsymmetrical induction motor.

$$[\underline{U}_S] = \begin{bmatrix} \underline{Z}_{A_+} & 0 & 0 \\ 0 & \underline{Z}_{A_-} & 0 \\ 0 & 0 & \underline{Z}_{A_0} \end{bmatrix} \cdot [\underline{I}_S]. \quad (35)$$

Using (34) and (35), we can write:

$$\begin{bmatrix} \underline{Z}_{A_+} & 0 & 0 \\ 0 & \underline{Z}_{A_-} & 0 \\ 0 & 0 & \underline{Z}_{A_0} \end{bmatrix} = [A]^{-1} \cdot [\underline{Z}] \cdot [A]\,. \quad (36)$$

After an iterative process on the right side of the equation, we obtain in the end:

$$\begin{aligned} \underline{Z}_{A_+} &= R + \mathrm{j}\,X_\sigma + \mathrm{j}\,X_{AA\sigma}(1-\beta) + h\underline{Z}_+ + \varepsilon\underline{Z}_- \\ \underline{Z}_{A_-} &= R + \mathrm{j}\,X_\sigma + \mathrm{j}\,X_{AA\sigma}(1-\beta) + \varepsilon\underline{Z}_+ + h\underline{Z}_- \\ \underline{Z}_{A_0} &= R + \mathrm{j}\,X_\sigma + \mathrm{j}\,X_{AA\sigma}(1+2\beta) + \\ &\quad + (1+2\beta)(\underline{Z}_+ + \underline{Z}_-) \end{aligned} \quad (37)$$

with notations:

$$\begin{aligned} R &= \frac{1}{3}\left(R_A + \frac{1}{k_B^2}R_B + \frac{1}{k_C^2}R_C\right) \\ X_\sigma &= \frac{1}{3}\left(X_{A\sigma} + \frac{1}{k_B^2}X_{B\sigma} + \frac{1}{k_C^2}X_{C\sigma}\right) \\ \alpha &= \frac{1}{3}\left(\sin\gamma_B - \sin\gamma_C - \sin(\gamma_B - \gamma_C)\right) \\ \beta &= \frac{1}{3}\left(\cos\gamma_B + \cos\gamma_C + \cos(\gamma_B - \gamma_C)\right) \\ h &= 1 - \beta + \sqrt{3}\alpha \\ \varepsilon &= 1 - \beta - \sqrt{3}\alpha \end{aligned} \quad (38)$$

Impedances $\underline{Z}_+$ and $\underline{Z}_-$ in relation (37) regarding phase A and motor parameters are related to the reference winding turn number, i.e. phase A.

According to these results, we may now assert the following: the induction motor with three unsymmetrical phases, supplied by the unbalanced sinusoidal voltages has been split in three fictitious symmetrical motors.

The first three-phase fictitious motor has the phase impedance $\underline{Z}_{A_+}$ and it is supplied by a positive sequence system of symmetrical voltages ($\underline{U}_{A_+}$, $\underline{U}_{B_+}$, $\underline{U}_{C_+}$). The second fictitious motor has the phase impedance $\underline{Z}_{A_-}$ and it is supplied by a negative sequence system of symmetrical voltages. Finally, the third fictitious motor has the phase impedance $\underline{Z}_{A_0}$ with a voltage system of zero sequence. Each sequence has distinct voltages, distinct currents and distinct motors. All of them are only fictitious and are obtained through mathematical computation.

Relation (37) suggests a simple equivalent circuit (figure 5) for each of the three fictitious symmetrical motors.

For the positive sequence the apparent power in complex form is:

$$\underline{S}_+ = \underline{U}_{A_+}\underline{I}^*_{A_+} + \underline{U}_{B_+}\underline{I}^*_{B_+} + \underline{U}_{C_+}\underline{I}^*_{C_+} =$$

$$= \underline{U}_{A_+}\underline{I}^*_{A_+} + k_B\underline{U}_{A_+}\mathrm{a}^2\left(\frac{1}{k_B}\underline{I}^*_{A_+}\mathrm{a}\right) +$$

$$+ k_C\underline{U}_{A_+}\mathrm{a}\left(\frac{1}{k_C}\underline{I}^*_{A_+}\mathrm{a}^2\right) = 3\underline{U}_{A_+}\underline{I}^*_{A_+}\,. \qquad (39)$$

Last part of this relation confirms the symmetry of the positive sequence fictitious motor represented in figure 6.a. For negative sequence we obtain:

$$\underline{S}_- = 3\underline{U}_{A_-}\underline{I}^*_{A_-} \qquad (40)$$

and the negative sequence motor is represented in figure 6.b. For the zero sequence:

$$\underline{S}_0 = 3\underline{U}_{A_0}\underline{I}^*_{A_0} \qquad (41)$$

and the diagram is the one in figure 6.c.

Impedances $\underline{Z}_{A_+}$, $\underline{Z}_{A_-}$, $\underline{Z}_{A_0}$ depend from the phase turn number and from the spatial displacement angle (γ_B and γ_C). In figure 7 and in figure 8 are represented the dependence of h and ε by the electrical displacement angle between phases.

DISCUSSIONS

- For a three-phase induction motor with following data: γ_B = 120°, γ_C = 240°, $k_B = k_C$ = 1 from relation (38) we obtain h = 3, ε = 0, β = -1/2, $R=R_A$, $X_\sigma = X_{A\sigma}$. With these values impedance from (37) becomes:

$$\underline{Z}_{A_+} = R_A + \mathrm{j}\left(X_{A\sigma} + \frac{3}{2}X_{AA\sigma}\right) + 3\underline{Z}_+\,. \qquad (42)$$

We can see that this is the particular case of a three-phase symmetrical motor, with the equivalent diagram known from figure 3.

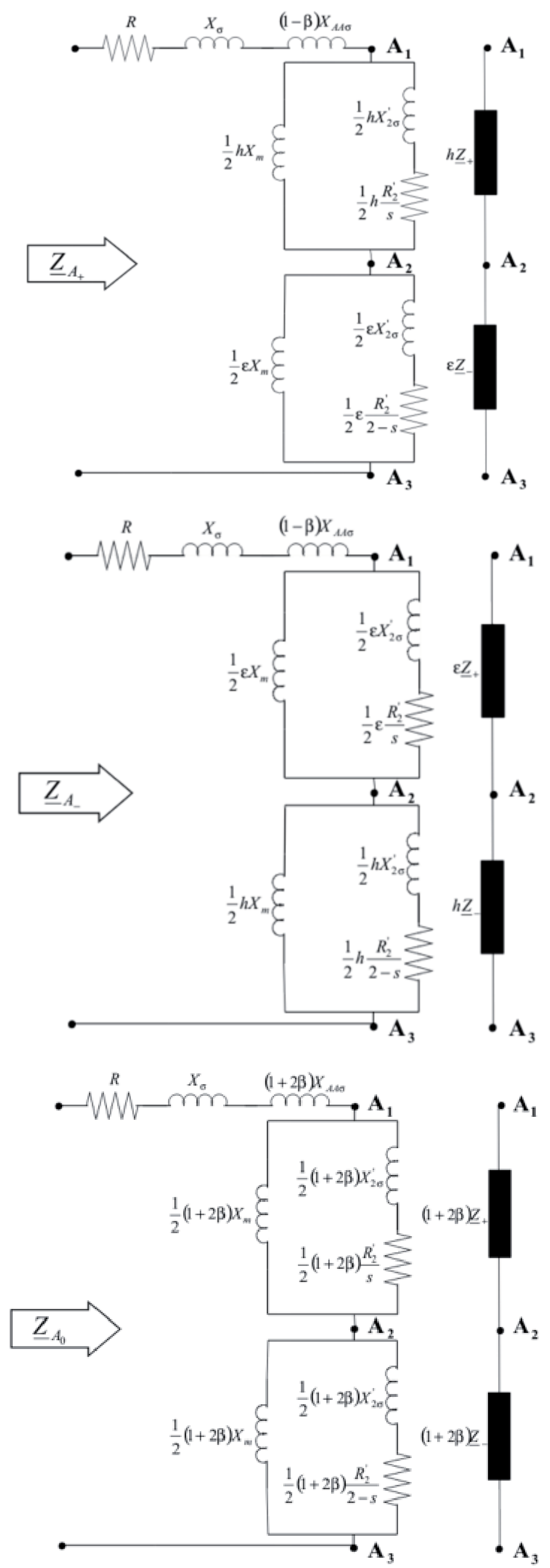


Fig. 5. The equivalent circuits of the fictitious symmetrical motors.

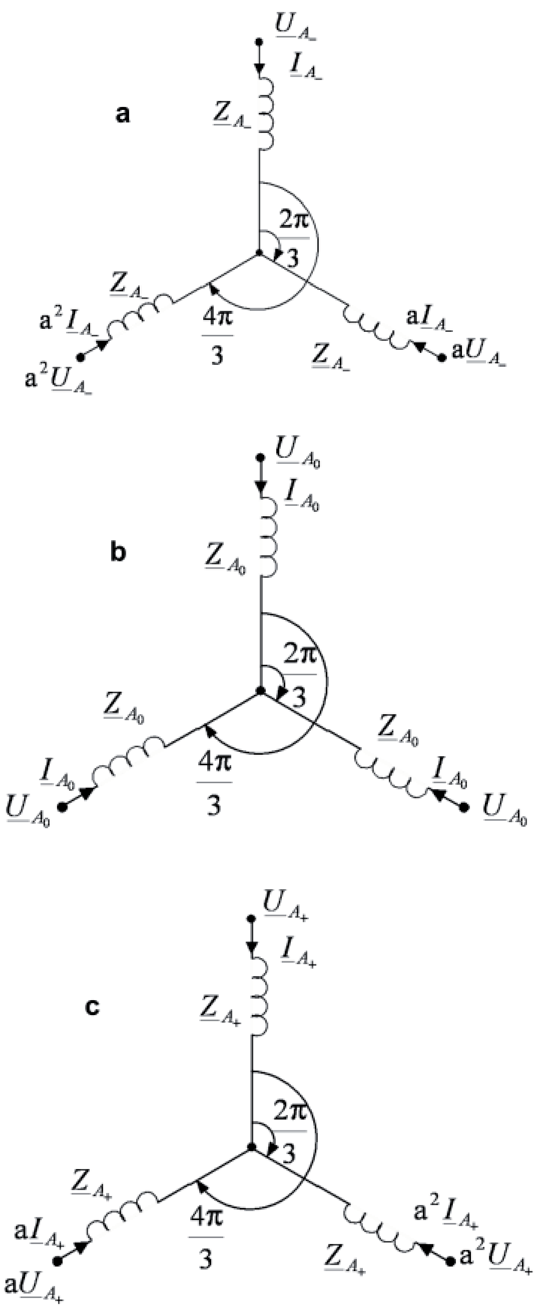


Fig. 6. Schematic representation of the three fictitious symmetrical motors.

- Any deviation from the “three-phase symmetry” leads to value of $\varepsilon > 0$, i.e. to a negative sequence component (for the magnetic field, for the current, for the torque, etc.), while the positive sequence decreases ($h < 3$). It is found once more that the best three-phase electrical machine is the symmetrical one, where $h = 3$ and $\varepsilon = 0$.

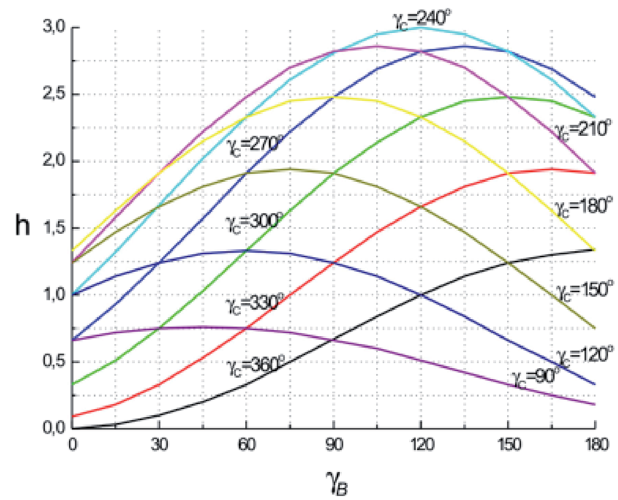


Fig. 7. Variation curves of coefficient *h* versus γ_B for different values of γ_C =constant.

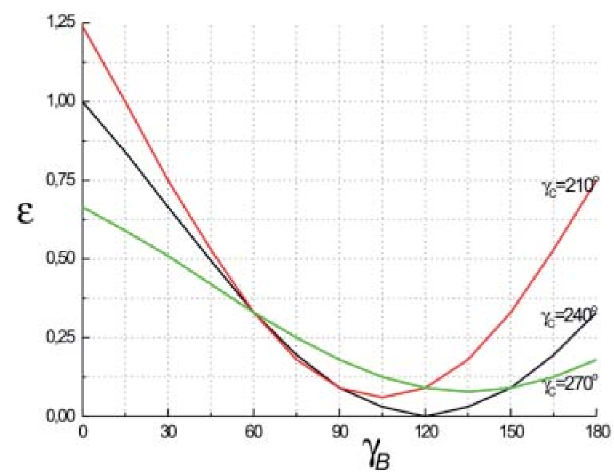


Fig. 8. Variation curves of coefficient ε versus γ_B for some values of fixed γ_C =0.

- From figures 7 and 8 we found that for γ_C = constant, any curve h = f(γ_B) has a maximum for $\gamma_B = \gamma_C/2$ and any curve ε = f(γ_B) has a minimum also for $\gamma_B = \gamma_C/2$. In other words, if the phases A and C are fixed, the best place for phase B is on the bisector of the angle γ_C.
- From relations (24) and (37) it can be seen that $\underline{Z}_{A_-}$ may be obtained from $\underline{Z}_{A_+}$ by replacing (s) with (2-s) we may write:

$$\underline{Z}_{A_-}(s) = \underline{Z}_{A_+}(2-s). \tag{43}$$

This means that fictitious motors in figure 6.a. and figure 6.b. are identical (have same constructive data). However, they have distinct impedances ($\underline{Z}_{A_+} \neq \underline{Z}_{A_-}$) related to the positive sequence and negative sequence of the supplied voltage, respectively.

CONCLUSIONS

The paper analyses the general case of the three-phase induction motor with unsymmetrical stator windings supplied by three-phase unbalanced voltages. The analyzed motor has a symmetrical cage in the rotor.

By using the known method of symmetrical components it has been analyzed the unbalanced voltage system. The expressions of the symmetrical components of supplying currents and voltages have been established.

Based on these symmetrical components, the expressions of the positive, negative and zero sequence impedances of the motor have been obtained. Such impedances represent three fictitious symmetrical motors supplied with positive, negative and zero sequence currents and voltages. By particularization of the elaborated mathematical expressions, the known cases of formulas and equivalent circuits in steady-state conditions have been finally obtained. Therefore, it is demonstrated that the best three-phase induction machine is the symmetrical one.

The obtained results allow the analysis of the influence of certain unsymmetrical stator windings on the squirrel-cage induction motor performances in steady-state conditions.

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